Logarithmic Differentiation / Review

1) $f(x) = 2\sin x \cos x$

$f'(x) = 2\sin x(-\sin x) + 2\cos x(\cos x)$

$f'(x) = -2\sin^2 x + 2\cos^2 x$

2) $s = \cot\left(\frac{2}{t}\right)$

$s' = -\csc^2\left(\frac{2}{t}\right)\left(-2t^{-2}\right)$

3) $r = \sec(1+3\theta)$

$r' = 3\sec(1+3\theta)\tan(1+3\theta)$

4) $y = \ln\sqrt{x} = \ln x^{1/2} = \frac{1}{2}\ln x$

$y' = \frac{1}{2}\left(\frac{1}{x}\right) = \frac{1}{2x}$

5) $y = e^{1+\ln x} = e \cdot e^{\ln x} = ex$

$\boxed{y' = e}$

6) $r = \log_2(\theta^2)$

$r' = \frac{2\theta}{\theta^2 \cdot \ln 2} = \frac{2}{\theta \ln 2}$

7) $y = x^{\ln x}$

$\ln y = \ln x^{\ln x}$

$\ln y = (\ln x)(\ln x)$

$\ln y = (\ln x)^2$

$\frac{y'}{y} = 2(\ln x)\cdot\frac{1}{x}$

$y' = 2\ln x\left(\frac{1}{x}\right)\cdot x^{\ln x}$

8) $f(x) = (\sin x)^x$

$\ln y = x \cdot \ln(\sin x)$

$\frac{y'}{y} = x \cdot \frac{\cos x}{\sin x} + \ln(\sin x)$

$y' = \left[\frac{x\cos x}{\sin x} + \ln(\sin x)\right]\cdot(\sin x)^x$

9) $f(x) = x \ln x$

$f'(x) = x\left(\frac{1}{x}\right) + \ln x$

$f'(x) = 1 + \ln x$

10) $xy + 2x + 3y = 1$

$xy' + y + 2 + 3y' = 0$

$xy' + 3y' = -2 - y$

$$\boxed{y' = \frac{-2-y}{x+3}}$$

11) $y^2 = \frac{x}{x+1}$

$2y \cdot \frac{dy}{dx} = \frac{x+1-x}{(x+1)^2}$

$$\boxed{\frac{dy}{dx} = \frac{1}{2y(x+1)^2}}$$

12) $\sqrt{xy} = 1$

$\frac{1}{2}(xy)^{-1/2}[xy' + y] = 0$

$xy' + y = 0$

$$\boxed{y' = -\frac{y}{x}}$$

13) $x^3 + y^3 = 1$

$3x^2 + 3y^2 \frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{x^2}{y^2}$

$\frac{d^2y}{dx^2} = \frac{y^2(-2x) - (-x^2)\left(2y\frac{dy}{dx}\right)}{y^4}$

$\frac{d^2y}{dx^2} = \frac{-2xy^2 + 2x^2y\left(-\frac{x^2}{y^2}\right)}{y^4}$

$\frac{d^2y}{dx^2} = \frac{-2xy^2 - \frac{2x^4}{y}}{y^4}$

$\frac{d^2y}{dx^2} = \frac{-2xy^3 - 2x^4}{y^5}$

14) $f(x) = x e^{\sin x}$

$f'(x) = x\cos x\, e^{\sin x} + e^{\sin x}$

$= e^{\sin x}\left[x\cos x + 1\right]$

$f''(x) = e^{\sin x}\left[-x\sin x + \cos x\right] + \cos x\, e^{\sin x}\left[x\cos x + 1\right]$

15) $f(x) = \sqrt{x^2 - 2x}$, $x = 3$

Point	Slope
$(3, \sqrt{3})$	$f'(x) = \frac{1}{2}(x^2-2x)^{-1/2}(2x-2)$
	$f'(3) = \frac{2}{\sqrt{3}}$

T: $y - \sqrt{3} = \frac{2}{\sqrt{3}}(x-3)$

N: $y - \sqrt{3} = -\frac{\sqrt{3}}{2}(x-3)$

16) $x + \sqrt{xy} = 6 \quad (4,1)$

$1 + \frac{1}{2}(xy)^{-1/2}(xy' + y) = 0$

$1 + \frac{1}{2}(4)^{-1/2}\left[4y' + 1\right] = 0$

$1 + \frac{1}{4}\left[4y' + 1\right] = 0$

$4y' + 1 = -4$

$y' = -\frac{5}{4}$

T: $y - 1 = -\frac{5}{4}(x-4)$

N: $y - 1 = \frac{4}{5}(x-4)$

17) a) $y = \sqrt{x} \cdot f(x)$

$y' = \sqrt{x} \cdot f'(x) + \frac{1}{2\sqrt{x}} f(x)$

$y'\big|_{x=1} = 1\left(\frac{1}{5}\right) + \frac{1}{2}(-3)$

$= \frac{1}{5} - \frac{3}{2}$

$\boxed{= -\frac{13}{10}}$

b) $y = f(1 - 5\tan x)$

$y' = f'(1-5\tan x)(-5\sec^2 x)$

$y'\big|_{x=0} = f'(1)(-5\sec^2 0)$

$= \left(\frac{1}{5}\right)(-5)$

$\boxed{= -1}$

c) $y = \frac{f(x)}{2+\cos x}, \quad x = 0$

$y' = \frac{(2+\cos x)\cdot f'(x) - f(x)(-\sin x)}{(2+\cos x)^2}$

$y'\big|_{x=0} = \frac{(3)\cdot f'(0) - 0}{9}$

$\boxed{= \frac{-6}{9} = -\frac{2}{3}}$